MODELLING OF ELECTROMAGNETIC WAVE USING STATISTICAL/NUMERICAL APPROACH

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by

Wan Najwati binti Wan Mahamad

Dissertation submitted in partial fulfilment of
the requirements for the
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JUNE 2009

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CERTIFICATION OF APPROVAL

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Approved:

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Pn. Hanita Bt. Daud,
Project Supervisor

UNIVERSITI TEKNOLOGI PETRONAS

TRONOH, PERAK

JUNE 2009

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

Wan Najwati binti Wan Mahamad

ABSTRACT

This paper reviews the problem statement, objectives, some background studies and methodology to be used in modelling Electromagnetic wave (EM wave) using the integrated programming language, MATLAB. This project is focusing on amplitude modelling of EM wave at frequency of 0.25 Hz using statistical and numerical approach. The significance of this project is that EM wave is one of the most recently used signal in communication and other technology nowadays. Therefore modelling of EM wave is widely required to do analysis and designing in computer. The objective of this project is to compare the EM wave models developed using statistical approaches and numerical approaches; in order to determine which method produce the best model. This estimated model is compared to the real data to determine the fitness of the model and to calculate the sum square error. The tools and equipment required in this project are computer and MATLAB software. The statistical approaches that are used in this project are first order and second order Regression Analysis. The numerical approaches used are Polynomial Curve Fitting and Spline Interpolation Technique. The coding of each method is saved in M-file to be executed in MATLAB. The results are observed and discussed from the resulting EM wave model using both approaches. At the end of this project, it can be seen that the numerical approach using Spline Interpolation Technique gave the most fitted model of the EM wave amplitude.

ACKNOWLEDGEMENT

Throughout the completion of my Final Year Project (FYP), I would like to acknowledge my family for always being there to provide moral support to me.

I would like to grant my greatest gratitude towards my Final Year Project supervisor, Pn.Hanita bt. Daud, and my co-supervisor, Dr. Vijanth Sagayan Asirvadam. During the completion of my FYP, they have provided lots of feedback that is related to the project as well as provided their valuable experiences and knowledge on the subject. They also consulted me on the aspects needed to do my FYP and the methods which are to be used to analyze the project so that the needed results can be obtained. Besides that, they always encouraged me to do the best and that the project can be solved by multiple methods and provide more alternatives for the project so that with more choices, chances that a better alternative can be found.

Last but not least, I would like to thank my housemates in Universiti Teknology PETRONAS for always being there when I need their helps in moral support and technical matter.

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CHAPTER 1

INTRODUCTION

1.1 Problem Statement

It is very important to introduce the concept of computational engineering to undergraduate study in the electromagnetic, as electromagnetic simulation has moved from analysis to design in engineering practice. Conventional analytical education in electromagnetic does not use computation and visualization techniques [9]. Also conventional methods cannot solve many real and practical engineering problems. Therefore, electromagnetic wave is required to be modeled in order to solve real practical problem and improve engineering design using computer. To solve such problems, a modeling of Electromagnetic wave (EM wave) using statistical and numerical approach in MATLAB is likely to be implemented.

1.2 Objectives and Scope of Study

The objectives of this project are:

1) To develop EM wave models using statistical approaches and analytical approaches
2) To compare estimated model to real data and calculate the error
3) To determine which method produce the best model

The scope of this project is modelling of EM wave based on the amplitude of EM wave (amplitude modelling) using the collected EM wave data at frequency of 0.25Hz from three (3) different receivers. The statistical approaches used in this project are the first order and second order Regression Analysis. The numerical approaches used are Polynomial Curve Fitting and Spline Interpolation Technique.

1.3 Background Study

1.3.1 Electromagnetic Waves

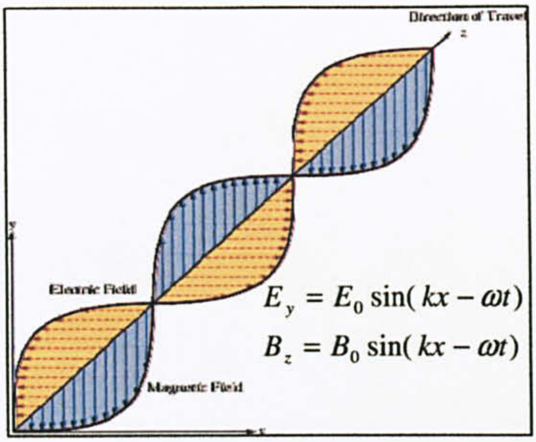

Figure 1: EM Wave model

Electromagnetic wave is a self-propagating wave where its components, Electric Field, E and Magnetic Field, B are orthogonal to each other and travel in a same direction[2]. Waves are characterized by frequency and wavelength:

$$v = f\lambda$$

Where v is the velocity of light, f is the frequency of EM wave and λ is the length of the wave. Electromagnetic wave is classified into types according to the frequencies and wavelength [8]:

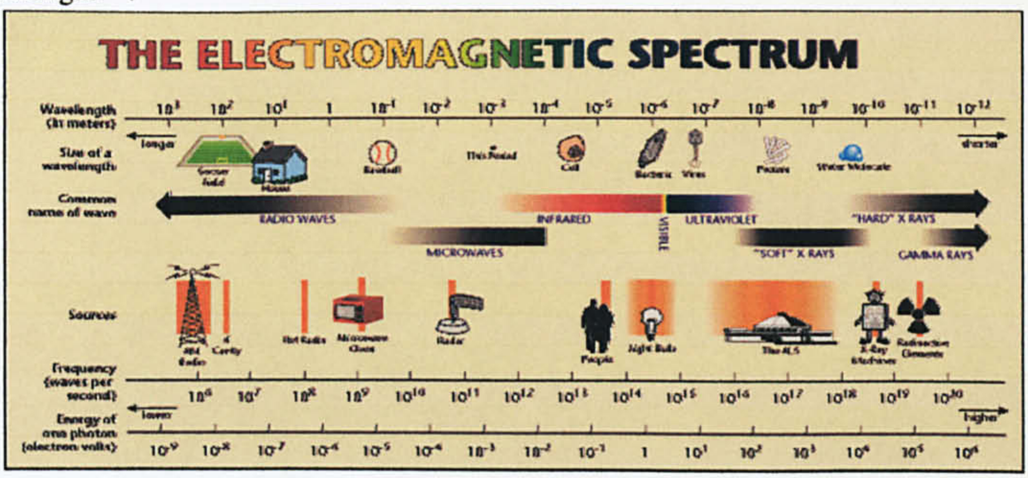

Figure 2: EM Waves Spectrum

In this project, the EM wave data is collected from three (3) different receivers as shown in Figure 3 below:

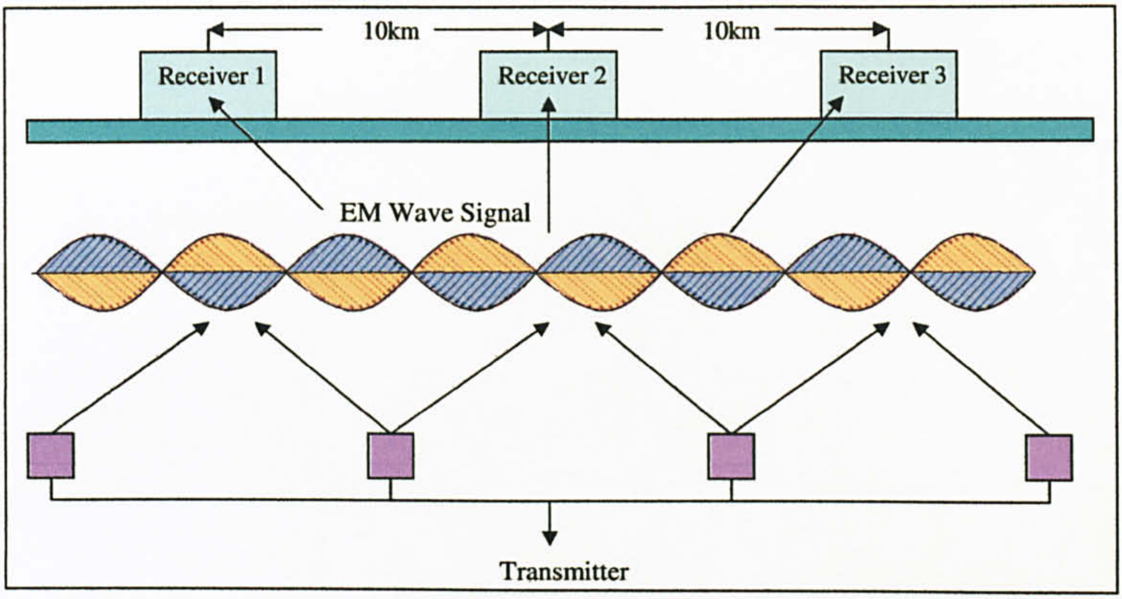

Figure 3: Capturing EM Wave Signal

EM wave signals are transmitted from the transmitters and captured by the three receivers. The captured signals are then sent to the computer and the available data of EM wave from the signals are extracted and managed using particular software.

CHAPTER 2

LITERATURE REVIEW

2.1 Homogeneous Equation

The electromagnetic wave equation is a second-order partial differential equation that describes the propagation of electromagnetic waves through a medium or in a vacuum. The homogeneous form of the equation, written in terms of either the electric field **E** or the magnetic field **B**, takes the form [1]:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E = 0 \quad \text{and} \quad \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)B = 0$$

Where:

c = speed of light in the medium. In a vacuum, $c = c_0 = 299{,}792{,}458$ m/s

B = Magnetic field

E = Electric field

t = Time

2.2 Speed of propagation

2.2.1 In vacuum

If the wave propagation is in vacuum, then the speed is[1]:

$$c = c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8{}_{(m/s)}$$

Where:

μ_0 = The magnetic constant

ε_0 = The vacuum permittivity

μ_0 and ε_0 are important physical constants that play a key role in electromagnetic theory. Their values (also a matter of definition) in SI units are tabulated below[8]:

Table 1: List of Symbols Used in Formulas and its Value

Symbol	Name	Numerical Value	SI Unit of Measure	Type
c_0	speed of light in vacuum	3.8×10^8	meters per second	**defined**
ε_0	electric constant	$8.854187817\times10^{-12}$	farads per meter	*derived*; $\frac{1}{\mu_0 c_0{}^2}$
μ_0	magnetic constant	$4\pi\times10^{-7}$	henries per meter	**defined**

2.2.2 In a material medium

The speed of light in a linear, isotropic, and non-dispersive material medium is[1]:

$$c = \frac{c_0}{n} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Where:

$$n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

n is the refractive index of the medium, μ is the magnetic permeability of the medium, and ε is the electric permittivity of the medium.

2.3 Electromagnetic wave equation in a vacuum

To obtain the electromagnetic wave equation in a vacuum using the modern method, we begin with the modern 'Heaviside' form of Maxwell's equations. In a vacuum, these equations are[1]:

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Taking the curl of the curl equations gives:

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times B = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla \times \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \nabla \times E = -\mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

By using the vector identity

$$\nabla \times (\nabla \times V) = \nabla (\nabla \cdot V) - \nabla^2 V$$

where V is any vector function of space, it turns into the wave equations[1]:

$$\frac{\partial^2 E}{\partial t^2} - {c_0}^2 \cdot \nabla^2 E = 0$$

2.4 Currently Used Numerical Method in Modelling

2.4.1 Finite-difference time-domain (FDTD)

Finite-difference time-domain **(FDTD)** is a popular computational electrodynamics modeling technique. It is considered easy to understand and easy to implement in software. Since it is a time-domain method, solutions can cover a wide frequency range with a single simulation run. The FDTD method belongs in the general class of grid-based differential time-domain numerical modeling methods[8]. Maxwell's equations (in partial differential form) are modified to central-difference equations and implemented in software. The equations are solved in a leapfrog manner: the electric field is solved at a given instant in time, then the magnetic field is solved at the next instant in time, and the process is repeated over and over again.

Since about 1990, FDTD techniques have emerged as primary means to model many scientific and engineering problems dealing with electromagnetic wave interactions with material structures. Current FDTD modeling applications range from near-DC (ultralow-frequency geophysics involving the entire Earth-ionosphere waveguide) through microwaves (radar signature technology, antennas, wireless communications devices, digital interconnects, biomedical imaging/treatment) to visible light [8].

2.4.2 Finite element method (FEM)

The finite element method (FEM) is used for finding approximate solution of partial differential equations (PDE) and integral equations. The solution approach is based either on eliminating the differential equation completely (steady state problems), or rendering the PDE into an equivalent ordinary differential equation, which is then solved using standard techniques such as finite differences, etc[8].

In solving partial differential equations, the primary challenge is to create an equation which approximates the equation to be studied, but which is numerically stable, meaning that errors in the input data and intermediate calculations do not accumulate and cause the resulting output to be meaningless. There are many ways of doing this, all with advantages and disadvantages.

2.4.3 Finite integration technique (FIT)

The finite integration technique (FIT) is a spatial discretization scheme to solve electromagnetic field problems in time and frequency domain numerically. It preserves basic topological properties of the continuous equations such as conservation of charge and energy. This method covers the full range of electromagnetics, from static up to high frequency and optic applications and is the basis for the commercial simulation tool CST Studio Suite ™ developed by Computer Simulation Technology (CST AG) [8].

The basic idea of this approach is to apply the Maxwell's equations in integral form to a set of staggered grids. This method stands out due to high flexibility in geometric modeling and boundary handling as well as incorporation of arbitrary material distributions and material properties such as anisotropy, non-linearity and dispersion. Furthermore, the use of a consistent dual orthogonal grid (e.g. Cartesian grid) in conjunction with an explicit time integration scheme leads to extremely high efficient algorithms referred to both computation time and memory requirements which are especially adapted for transient field analysis in Radiofrequency (RF) applications.

2.5 Statistical Functions in MATLAB

One of the advantages of working in the MATLAB language is that functions operate on entire arrays of data, not just on single scalar values. The functions are said to be *vectorized.* Vectorization allows for both efficient problem formulation, using array-based data, and efficient computation, using vectorized statistical functions. When statistical functions operate on a matrix of numerical data, they treat the columns independently, as separate measured variables[9].

MATLAB statistical functions expect data input arguments to be in the form of numerical arrays. If data is stored in a cell or structure array, it must be extracted to a numerical array, via indexing, for processing. Statistics Toolbox functions are more flexible[5]. Many toolbox functions accept data input arguments in the form of both numerical arrays and dataset arrays, which are specifically designed for storing general statistical data[9].

MATLAB data containers (variables) are suitable for completely homogeneous data (numeric, character, and logical arrays) and for completely heterogeneous data (cell and structure arrays) [9]. Statistical data, however, are often a mixture of homogeneous variables of heterogeneous types and sizes. Dataset arrays are suitable containers for this kind of data. Dataset arrays can be viewed as tables of values, with rows representing different observations or cases and columns representing different measured variables[9].

2.6 Linear Regression Model

In statistics, linear regression models often take the form of[5]:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1{}^2 + \beta_5 x_2{}^2 + \varepsilon$$

Here, a response variable *y* is modeled as a combination of constant, linear, interaction, and quadratic terms formed from two predictor variables x_1 and x_2. Uncontrolled factors and experimental errors are modeled by ε. Given data on x_1 and x_2, and *y*, regression estimates the model parameters β*j* (*j* = 1, ..., *5*). More general linear regression models represent the relationship between a continuous response *y* and a continuous or categorical predictor *x* in the form of[5]:

$$y = \beta_1 f_1(x) + ... + \beta_j f_j(x) + \varepsilon$$

The response is modeled as a linear combination of (not necessarily linear) functions of the predictor, plus a random error ε [5]:. The expressions *fj*(***x***) (*j* = 1,..., *p*) are the *terms* of the model. The β*j* (*j* = 1, ..., *p*) are the *coefficients*. Errors ε are assumed to be uncorrelated and distributed with mean 0 and constant (but unknown) variance [5].

Examples of linear regression models with a scalar predictor variable *x* include[9]:

- Linear additive (straight-line) models
- Polynomial models
- Chebyshev orthogonal polynomial models
- Fourier trigonometric polynomial models

Examples of linear regression models with a vector of predictor variables *x* = *(x_1, ..., x_N)* include[9]:

- Linear additive (hyperplane) models
- Pairwise interaction models
- Quadratic models
- Pure quadratic models

Given n independent observations $(x_1, y_1), \ldots, (x_n, y_n)$ of the predictor x and the response y, the linear regression model becomes an n-by-p system of equations[5]:

$$\underbrace{\begin{pmatrix} y_1\\ \vdots \\ y_n\end{pmatrix}}_{y} = \underbrace{\begin{pmatrix} f_1(x_1) & \cdots & f_p(x_1)\\ \vdots & \ddots & \vdots\\ f_1(x_n) & \cdots & f_p(x_n)\end{pmatrix}}_{X}\underbrace{\begin{pmatrix}\beta_1\\ \vdots\\ \beta_p\end{pmatrix}}_{\beta} + \underbrace{\begin{pmatrix}\varepsilon_1\\ \vdots\\ \varepsilon_n\end{pmatrix}}_{\varepsilon}$$

X is the de X are the terms of the model evaluated at the predictors. To fit the model to the data, the system must be solved for the p coefficient values in $\beta = (\beta1, \ldots, \beta p)T$. The MATLAB backslash operator (\) solves systems of linear equations. Ignoring the unknown error ε , MATLAB estimates model coefficients in $y = X\beta$ using *betahat* = X\y where X is the design matrix and y is the vector of observed responses. MATLAB returns the least-squares solution to the system; the least-squares estimator *betahat* minimizes the norm of the residual vector y-X*beta over all beta. If the system is consistent, the norm is 0 and the solution is exact. In this case, the regression model interpolates the data. In more typical regression cases where $n > p$ and the system is overdetermined, the least-squares solution estimates model coefficients obscured by the error ε[5].

CHAPTER 3

METHODOLOGY

3.1 Tools and equipment

Table 2: The Tools and Equipment Required for the Project

Tools/Equipment	Requirement	Description
MATLAB 7.1 software	Statistic Toolbox	Execute and do the computation and statistical analysis of the selected EM data
	Basic Fitting Toolbox	Execute and do the computation of n^{th} order polynomial to fit a model
Laptop	Minimum requirement: - 256 MB of RAM - 1.6GHz processor	Laptop must have the minimum requirements to be able to run simulation on MATLAB

3.2 Research and Background Study

I have returned to my theoretical textbooks to seek out and understand the analytical solutions and to interpret complex equations and the nature of EM fields. I also learn how to use MATLAB Statistics Toolbox to implement the EM wave model and understand the concept of Regression Analysis, which is the statistical method that I am be using in modelling the EM wave.

Regression Analysis is a method used to fit the model to the data. It is used for the modelling and analysis of numerical data which consists of a **dependent variable Y** (response variable) and **independent variable X** (predictors) [5]. The **unknown parameters,**

beta and denoted as ***b*** is used to get the best fit of the model. There are a few of Regression Model. For the first step in modelling, I will be using **linear regression techniques** to find functions that illustrate the relationship among variables because they able to build mathematical models of the EM wave data set.

-The general form of the multiple regression equation is:

$$Y' = a + b_1x_1 + b_2x_2 + b_3x_3 + ... + b_nx_n$$

Where:

Y' = The predicted output of Regression (estimated output)

a , b = Regression coefficient

x = inputs

Generally, more predictor (input, x) will yield a better and more accurate output, Y' [5].

3.3 Modelling the EM Waves from the Raw Data

3.3.1 Data Collection and Gathering

A set of data of EM wave that have been collected from three (3) receivers are used to model the EM wave amplitude *(Refer to Appendix B).* The EM wave data consists the values of certain variables; accordingly to eight (8) frequencies which are 0.25Hz, 0.5Hz, 0.75Hz, 1.0Hz, 1.25Hz, 1.5Hz, 1.75Hz, 2.5Hz. The variables that are taken into consideration from the raw data are **frequency, magnitude, phase and range** of the EM wave. These variables will be analyzed using Regression Analysis in MATLAB to see the relationships between them. However, only few of the frequencies will be analyzed in this project. The frequencies which are used in the modelling in this project (randomly chosen) are 0.25Hz and 1.25Hz.

3.3.2 Linear Regression Analysis of the Data

Regression Analysis in MATLAB is executed through M-files. In this project, the Regression Analysis that has been performed was using the first order and the second order regression analysis. This method is chosen with consults from my co-supervisor.

In this project, the variables used to apply statistical analysis on the EM model are as below:

1)First-Order Multiple Regression Analysis

-Estimated Output, Y' = Amplitude of EM wave

-Input, x_1 and x_2 = Frequency and phase of EM wave

2)Second-Order Multiple Regression Analysis

-Estimated Output, Y' = Amplitude of EM wave

-Input, x_1, x_2 and x_3 = Frequency, phase and range of EM wave

The parameters a and b are the regression coefficients .

3.3.3 Polynomial Curve Fitting

Polynomial Curve Fitting will be done using Basic Fitting Tool in MATLAB. This tool will automatically generate the plot of the model. It also will compute and return the value of polynomial coefficient from first degree to tenth degree of polynomial model. The general form of polynomial is:

$$Y' = a + b_1x_1 + b_2x_2 + ... + b_nx_n$$

3.3.4 Spline Interpolation Technique

The basic idea of Spline Interpolation Technique is that it will try to generate functions between the knots that we randomly choose from a sorted data. It is a piecewise polynomial function, where different sections of polynomials are fitted smoothly together. The data involve are frequency, phase, and amplitude of the EM wave. The general form of Spline Equation is:

$$\sum_{j=1}^{n} B_{j,k}(x) = 1 \quad \text{on } [t_k..t_{n+1}]$$

Where $B_{j,k}$ is the piecewise-polynomial of degree $<k$, with knots t.

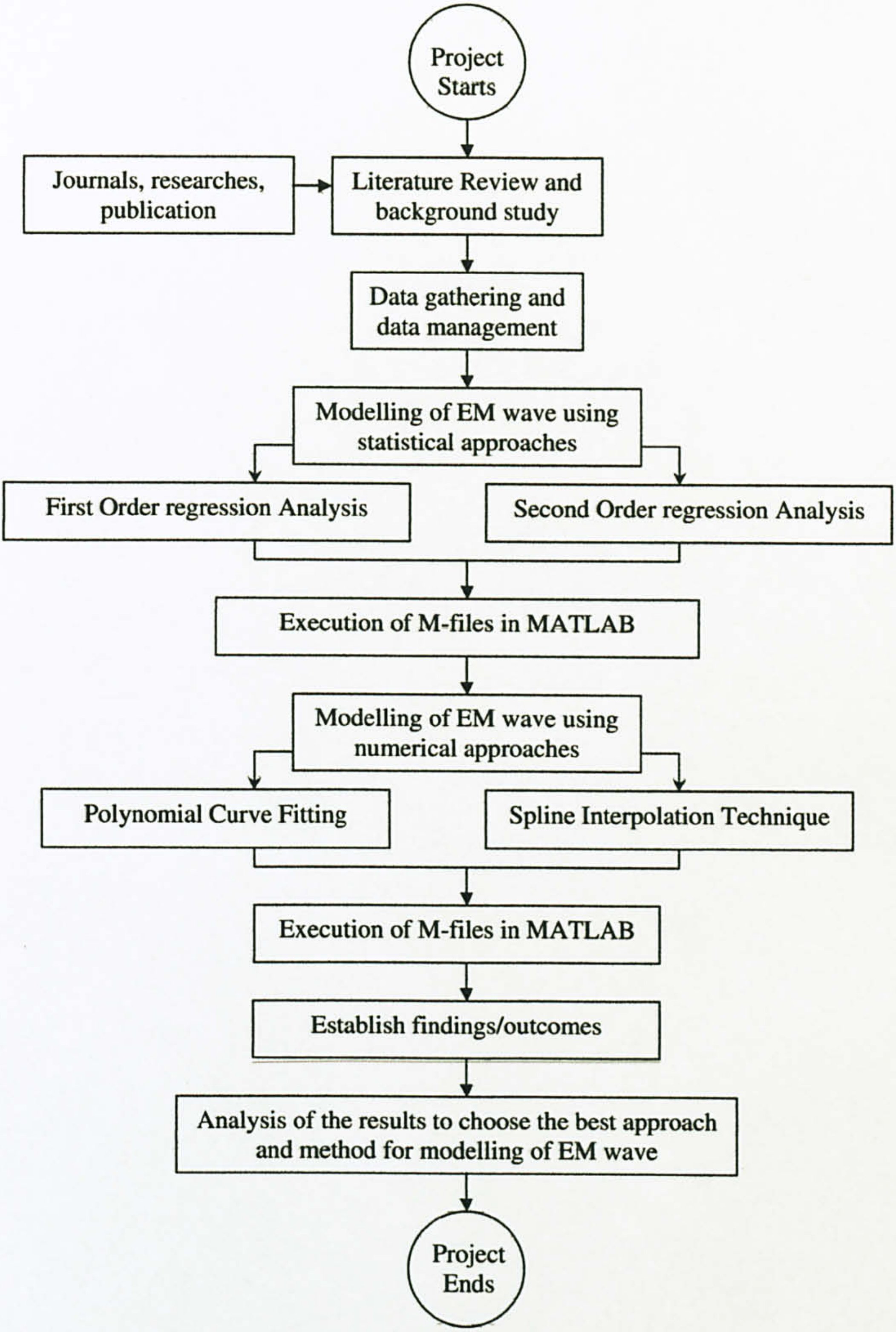

Figure 4: Flowchart of Overall Project

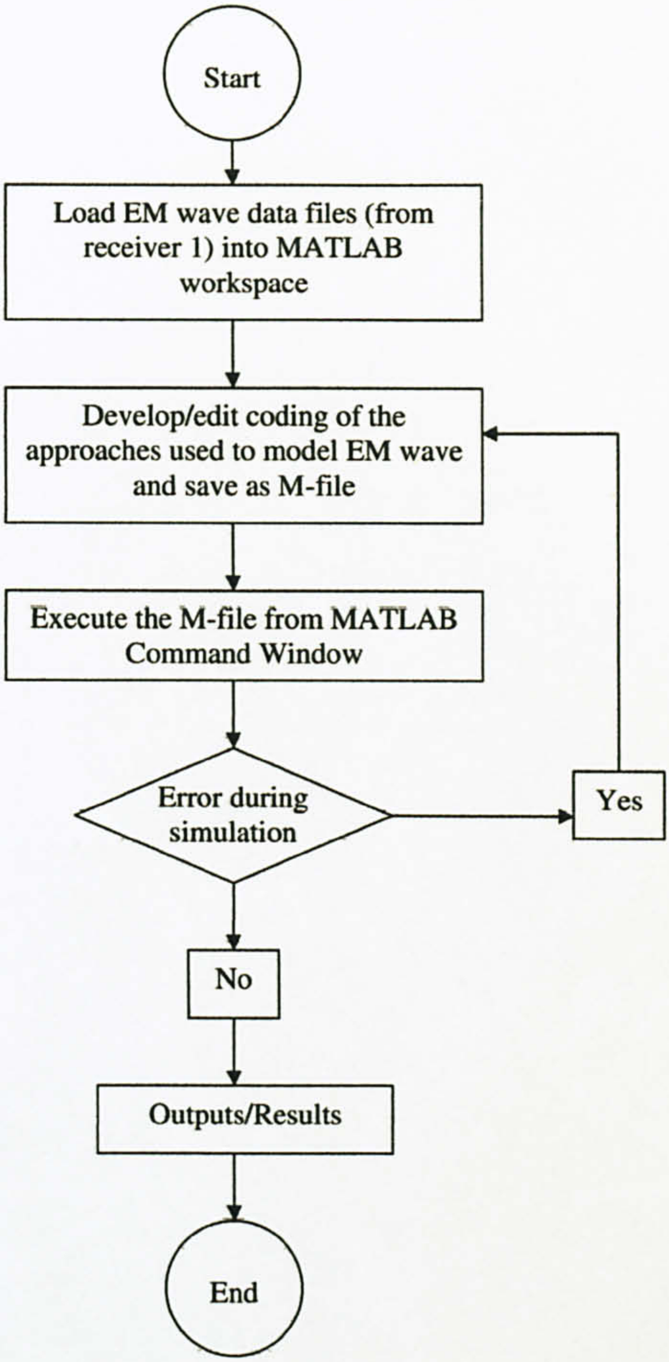

Figure 5: Flowchart of Simulation in MATLAB

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Results

4.1.1 First Order Regression Analysis

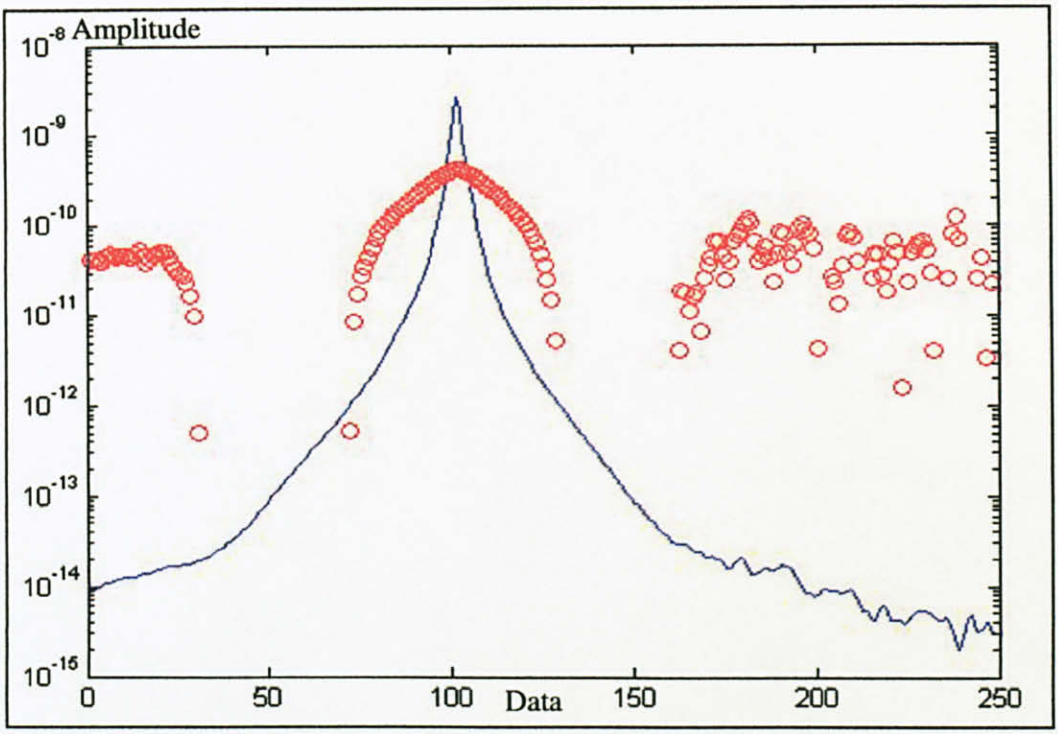

Figure 6: Estimated Model from the First Order Regression Analysis

Figure 6 shows the resulting graph when executing the coding developed in M-file *(Refer to Appendix A)*. The resulted plot is shown in 'log' scale at y-axis. The code performs first order regression analysis to the loaded data set. The data set contains 248 observations of two input variables, x (frequency and phase of the magnetic field). The blue lines are the real data while the red circles are the estimated model. The figure shows that the estimated output did not follow the pattern of the real data as expected.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Results

4.1.1 First Order Regression Analysis

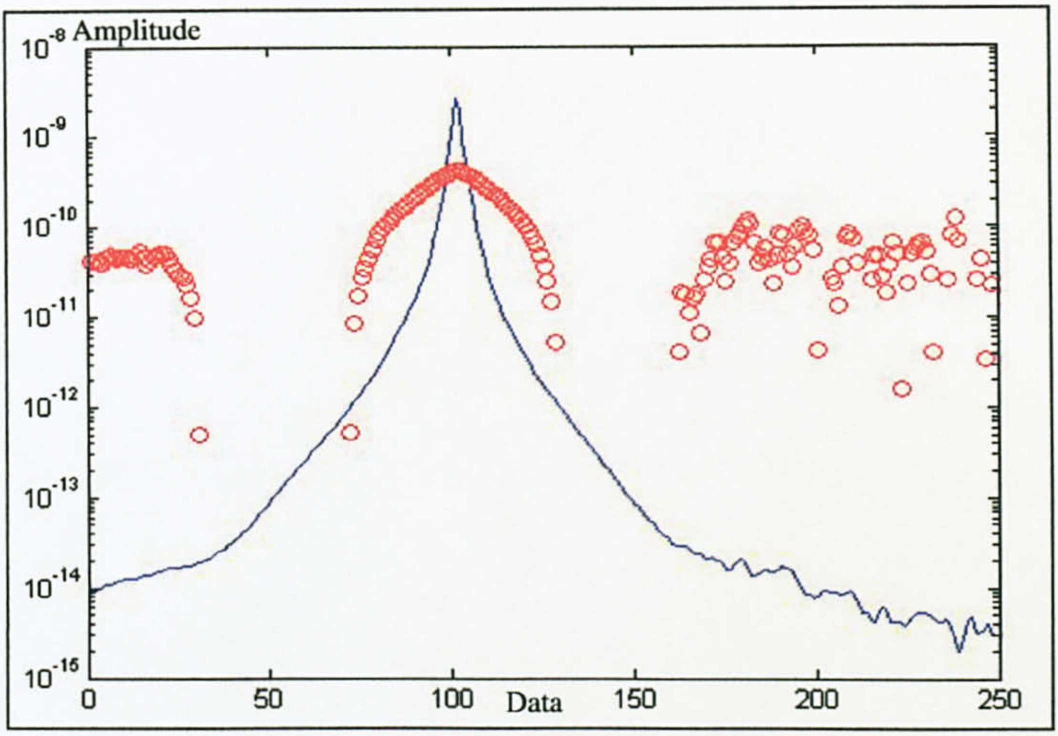

Figure 6: Estimated Model from the First Order Regression Analysis

Figure 6 shows the resulting graph when executing the coding developed in M-file *(Refer to Appendix A)*. The resulted plot is shown in 'log' scale at y-axis. The code performs first order regression analysis to the loaded data set. The data set contains 248 observations of two input variables, *x* (frequency and phase of the magnetic field). The blue lines are the real data while the red circles are the estimated model. The figure shows that the estimated output did not follow the pattern of the real data as expected.

4.1.2 Second Order Regression Analysis

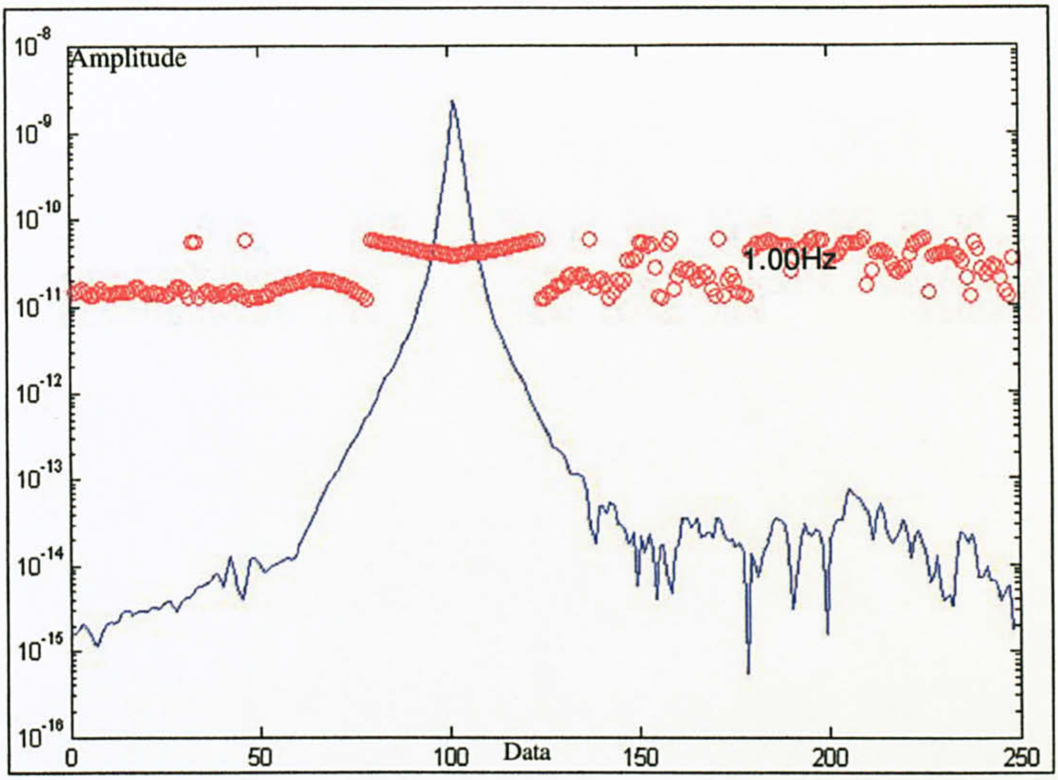

Figure 7: Estimated Model from the Second Order Regression Analysis

Figure 7 above shows the plot of data set containing 248 observations of the three input variables which are frequencies, phase and range of the EM wave. The resulted plot is shown in 'log' scale at y-axis. The estimated model (red circles) from second regression analysis is not fitted with the real data (blue lines). The model did not follow the pattern of real data at all.

From the results in Figure 6 and Figure 7, we can say that first order Regression Analysis give a batter model compared to the second order Regression Analysis. Therefore, the second order Regression Analysis method is eliminated and would not be used in the modelling process.

4.1.3 Polynomial Curve Fitting

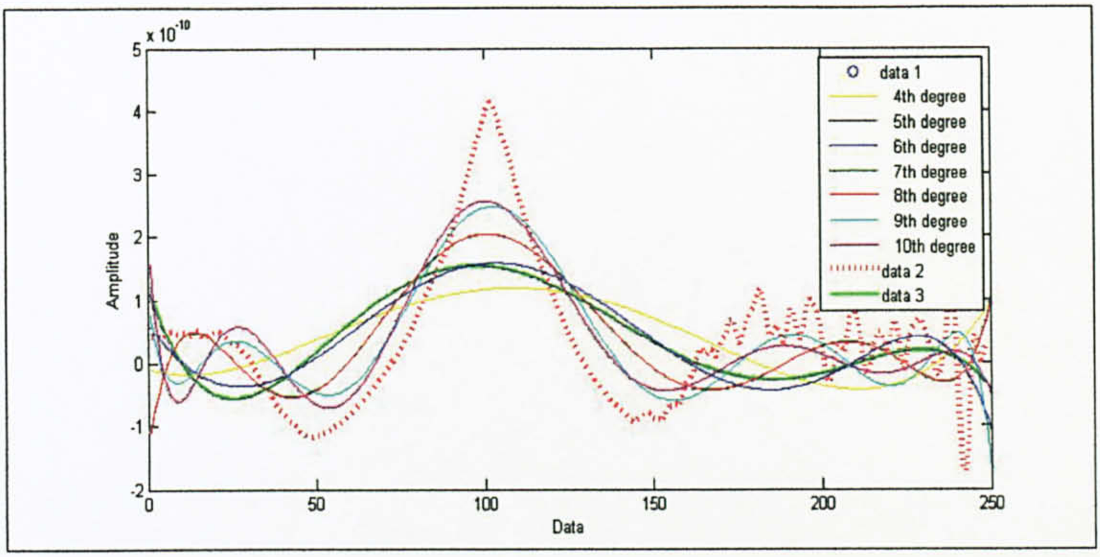

Figure 8: Comparison of Estimated Models Using Polynomial Curve Fitting and First Order Regression Analysis

Figure 8 shows that the new fit model using polynomials of order four (4) to ten (10) are not following the trend of the real output. The resulting plot is also shown in 'log' scale at y-axis. From Figure 8 it is observed that the polynomial curves (lines) are very near to the first order regression model (dotted line) which is not really a good fit to the real data. It also can be observe that as the order of polynomial increases, the lines will go nearer to the dotted line.

4.1.4 Spline Interpolation Technique

Spline Interpolation Technique is used to obtain the estimated model in blue line as shown in Figure 9. The blue line shows that the Spline model fit the real data very well. Using Basic Fitting Tool in MATLAB, the comparison between statistical approach and numerical approaches can be observed *(see Figure 9)*. The statistical approach used is first order Regression Analysis, while the numerical approaches used are Polynomial Curve Fitting and Spline Interpolation Technique. The second order Regression Analysis is eliminated because the estimated model is unacceptable.

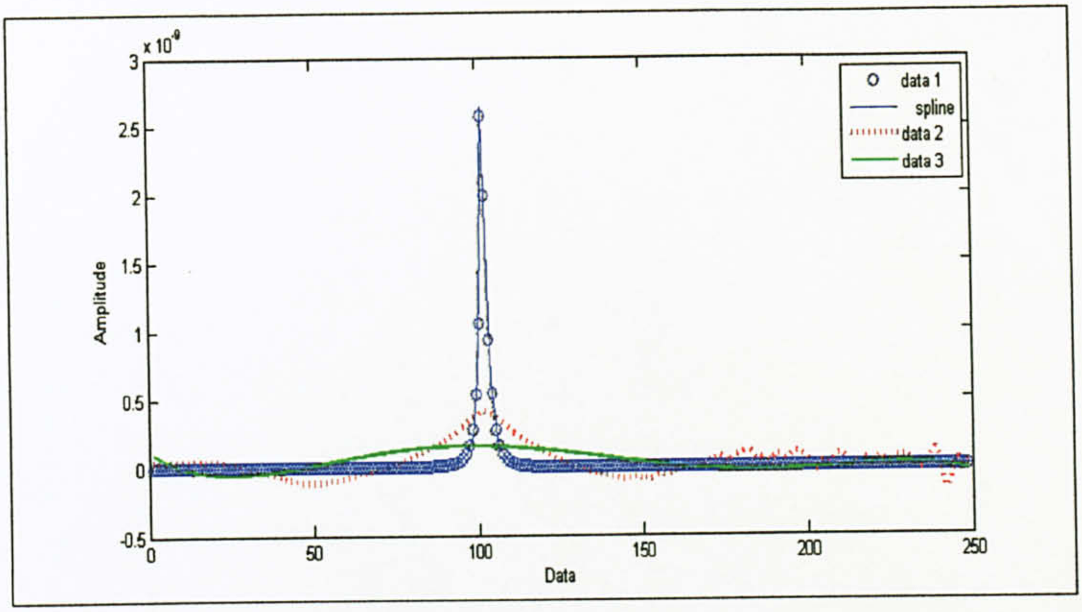

Figure 9: Comparison of Estimated Models Using Statistical/Numerical Approaches

Since the Spline Interpolation Technique obtain the expected model for the EM wave amplitude, therefore this method is used for further EM wave amplitude modelling and testing process. The testing is done to observe whether the model fit with the real data from other receivers and do the comparison by calculating the error of the model to the real data.

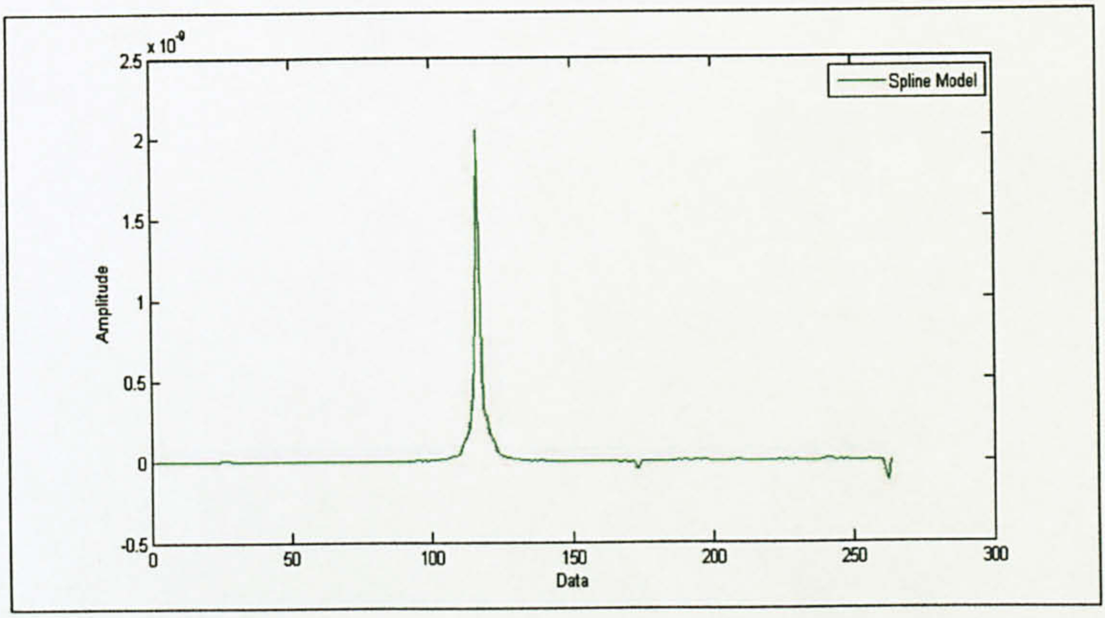

Figure 10: Spline Model of EM Wave Amplitude Using Data from Receiver 1

4.1.5 Testing of the Accepted Model (Spline Model)

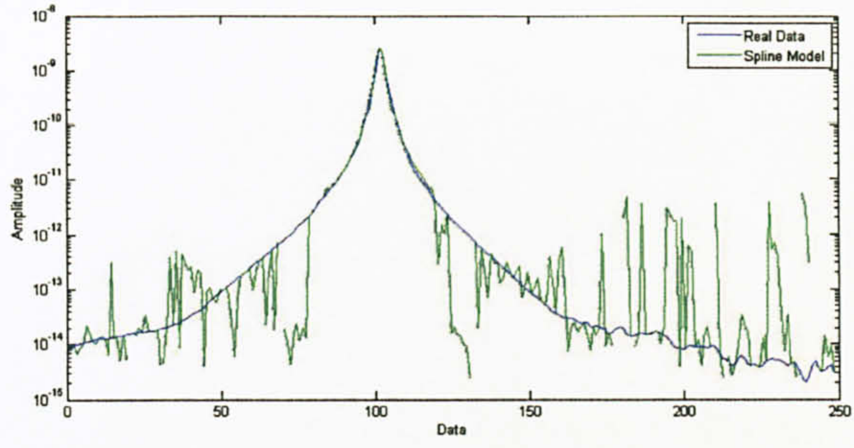

Figure 11: Comparison of the Model with the Real Data from Receiver 1

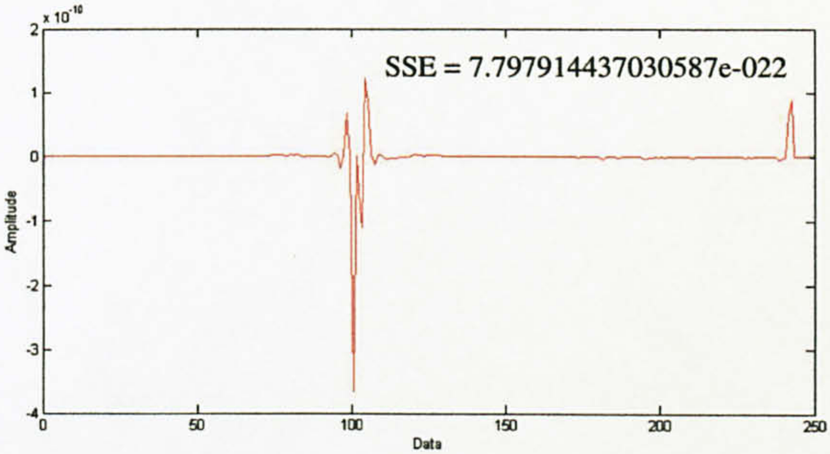

Figure 12: Error Plot of Model to Real Data from Receiver 1

The Sum Square Error (SSE) is calculated in MATLAB using the formula:

SSE = mean [(Model Amplitude – EM Wave Amplitude) ^{2}]

SSE of this model is very small when compared to the real data from Receiver 1 (7.797914437030587e-022) which is almost equal to zero.

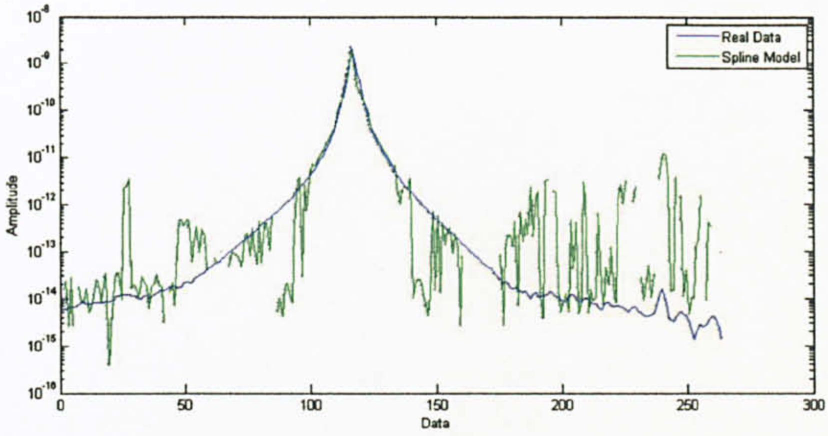

Figure 13: Comparison of the Model with the Real Data from Receiver 2

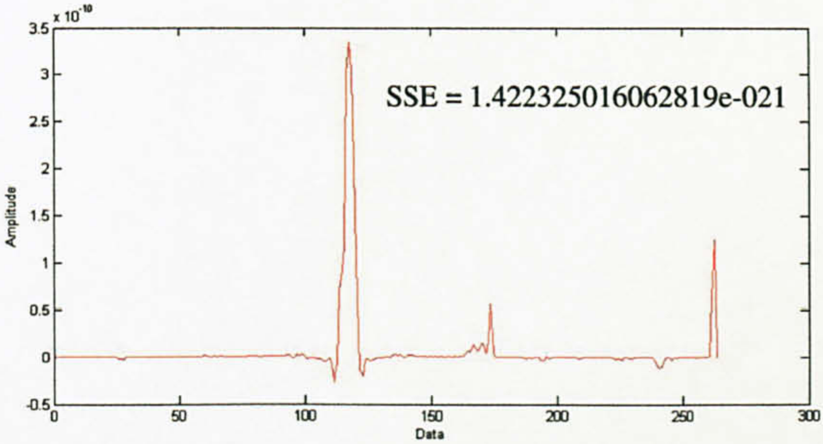

Figure 14: Error Plot of Model to Real Data from Receiver 2

The SSE of the model is very small when compared to the real data from Receiver 2 (1.422325016062819e-021) which is almost equal to zero. However, this value of SSE is bigger if compared to the SSE from Figure 12.

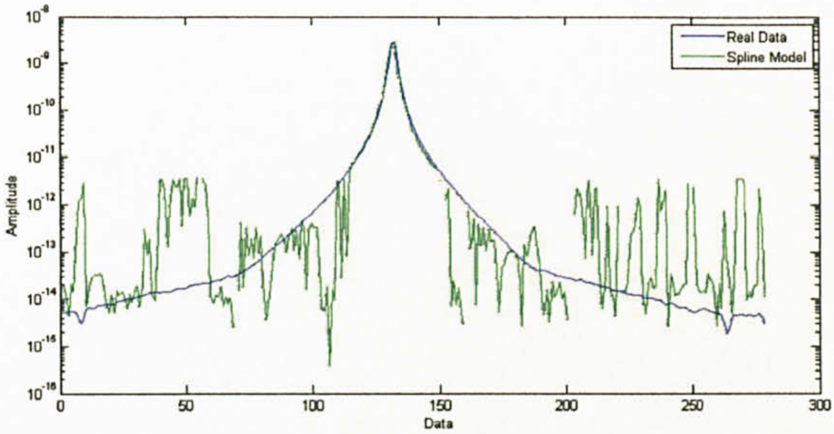

Figure 15: Comparison of the Model with the Real Data from Receiver 3

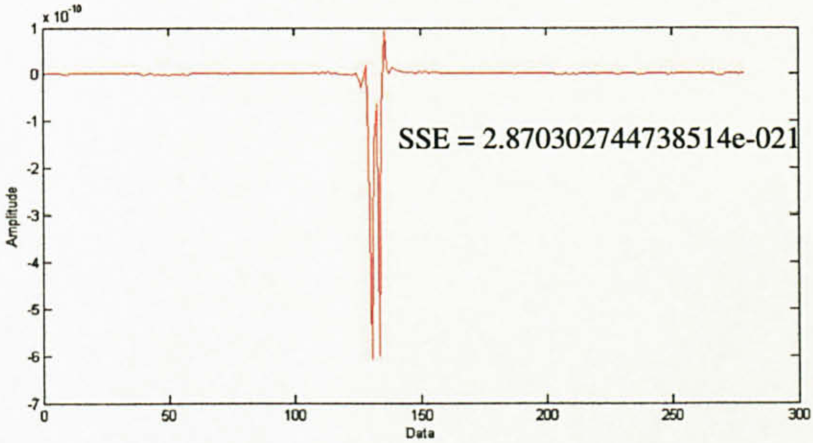

Figure 15: Error Plot of Model to Real Data from Receiver 3

Table 3: SSE of Model in Comparison to the Real Data from Three (3) Receivers

Receiver	**1**	**2**	**3**
SSE	7.797914437030587e-022	1.422325016062819e-021	2.870302744738514e-021

4.2 Discussion

Most of the resulting plots are shown in 'log' scale at y-axis so that the shape of the model and the real data can be seen clearer *(see Figure 6,7,8,11,13 and15).* If the plot is shown in normal scale, the only thing that can be seen is a spike of the amplitude.

Both of Regression Analysis results show that the estimated models are not fitted to the real data. However, it can be seen that the estimated model when using first order Regression Analysis is better than the estimated model when using second Regression Analysis *(see Figure 6 and Figure 7).* This is because the first order Regression Analysis shows that the estimated model try to achieve the shape of the real data at the peak *(see Figure 6).* While for the second order Regression Analysis model, it does not follow the shape of the real data at all *(see Figure 7).*

Polynomial Curve Fitting (numerical approach) also does not show the desired and expected model for the EM wave amplitude. This is because the model curves generated using this method is very far away from the shape of the real data *(see Figure 9).* During the fitting process in MATLAB, there are some data points are excluded. Those points are the points resulting from the large spike in the middle of the amplitude data *(see Figure 8).* Polynomial Curve Fitting technique treats those points as outliers and excludes them in the interpolation process. That is why the estimated model did not follow the shape of the real data.

The Spline Interpolation Technique (numerical approach) generates the most fitted model for the EM wave amplitude. This method produces a model with very similar shape to the real data *(see Figure 9).* When the Spline model *(see Figure 10)* is tested with the real EM wave amplitude data from Receiver 2 and Receiver 3, the resulting error is very small (almost equal to zero). Since the Spline model is generated using the data from Receiver 1, the resulting Sum Square Error (SSE) of the model when compared to the data from Receiver 1 is the smallest which is equal to 7.797914437030587e-022. The other two SSE for data from Receiver 2 and Receiver 3 are 1.422325016062819e-021 and 2.870302744738514e-021 respectively. The SSE is changing because different receiver would give different set of values in each variable due to difference of proximity to the EM wave.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

As a conclusion, I have achieved the objectives of this project which are to compare EM wave models developed using statistical approaches and analytical approaches and calculate the error. The comparison will lead to the decision on which method produce the best model. From the comparison and analysis that have been shown in Results and Discussion, it can be concluded that the best approach to model the EM wave amplitude is using numerical approach which apply the Spline Interpolation Technique. This is because Spline model showed the most fitted model to the real data with very small error which is almost zero.

At the end of this project, it also can be conclude that modelling of EM wave using statistical approach or numerical approaches is very easy to be implemented in MATLAB since no lengthy derivation of derivative formulas involves. This is because MATLAB is very powerful software in modelling using data sets and arrays.

5.2 Recommendation

The model obtained using Regression Analysis is not fit to the EM wave amplitude data. Therefore it is suggested that more modelling at different frequencies should be done using this technique to observe if there is any difference. If available, more variables also should be included and increase the order of the Regression Analysis to have a desired model.

In this project, the data used in modelling the EM wave amplitude is only from one receiver (Receiver 1). It is recommended that data from different receiver be used in the modelling and do some comparison and analysis to determine which data sets could produce a model with the least error.

REFERENCES

[1] Z.Haznadar and Z.Stih, *Electromagnetic Fields,Waves and Numerical Methods* (IOS Press Ohmsha, 2000)

[2] Jacques Lavergnat, Michel Sylvain, *Radio Wave Propagation: Principles and Techniques*, (John Wiley & Sons, 2000)

[3] Markus Zahn, *Electromagnetic Field Theory: a problem solving approach*, (John Wiley & Sons, 1997)

[4] Mosteller, F. and J. Tukey, *Data Analysis and Regression*, Addison-Wesley,1977.

[5] Chatterjee, S., and A. S. Hadi. "Influential Observations, High Leverage Points, and Outliers in Linear Regression," Statistical Science, 1986.

[6] Belsley, D. A., E. Kuh, R. E. Welsch, *Regression Diagnostics*, Wiley,1980.

[7] Karl F. Warnick,"An Intuitive Error Analysis for FDTD and Comparison to MoM," *IEEE Antennas and Propagation Magazine*, Vol.47, No.6, pp.111-115, Dec. 2005.

[8] T. Weiland, *A Discretization Method for the Solution of Maxwell's Equations for Six-Component Fields, Electronics and Communications AEUE*, vol. 31,1977.

[9] http://www.mathworks.com/products/matlab/

[10] http://ieeexplore.ieee.org/

[11] http://www.sciencedirect.com/

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APPENDICES

Appendix A: M-file of Coding for Modelling the EM Wave using Regression Analysis, Polynomial Curve Fitting and Spline Interpolation Technique

Appendix B: A Sample of EM Wave Used in the Modelling

1) First Regression Analysis for Each Frequency:

```
format long
file = load('filename.txt');
F = file(:,1);
Amp = file(:,3);
P = file(:,5);
R = file(:,12);
C0 = ones (248,1);
X = [C0 F P];
A = pinv (X) * Amp;
yp = A(1) + A(2) * F + A(3)*P;
semilogy (Amp)
hold
semilogy (yp,'r:')
```

2) Second Regression Analysis for All of the Eight Frequency

```
format long
file = load('filename.txt');
F = file(:,1);
Amp = file(:,3);
P = file(:,5);
R = file(:,12);
C0 = ones (248,1);
X = [C0 F P R];
A = pinv (X) * Amp;
yp = A(1) + A(2) * F + A(3)*P + A(4)*R;
semilogy (Amp)
hold
semilogy (yp,'r:')
```

3) Polynomial Curve Fitting Technique

```
format long
file = load('filename.txt');
F = file(:,1);
Amp = file(:,3);
P = file(:,5);
R = file(:,12);
C0 = ones(248,1);
X = [C0 F P];
A = pinv(X)*Amp;
fit = X * A;
n=248
[p,S]=polyfit(1:n,Amp',5)
newfit = polyval(p,1:n,S)
plot (1:n,Amp,'bo',1:n,fit,'r:',1:n,newfit,'g')
```

4) Spline Interpolation Technique

```
format long
APR = load('a_APR.txt');
Y = APR(:,1);
X = APR(:,2);
s0 = [X Y];
s1 = sortrows(s0);
s2 = s1(1:4:744,:);
x = s2(:,1);
y = s2(:,2);
xx = s0(:,1);
y1 = s0(:,2);
yy = spline(x,y,xx);
plot ([y1 yy])
figure
plot ([y1 - yy])
SSE = mean((y1-yy).^2)
```

5) Testing the Spline Model

```
format long
APR = load('a01_025.txt');
testdata = load('a03_025.txt');
t1 = testdata(:,3);
t2 = testdata(:,5);
Y = APR(:,3);
X = APR(:,5);
s0 = [X Y];
s1 = sortrows(s0);
s2 = s1(1:4:248,:);
x = s2(:,1);
y = s2(:,2);
xx = s0(:,1);
y1 = s0(:,2);
yy = spline(x,y,t2);
plot ([t1 yy])
SSE = mean((t1-yy).^2)
```

1) Data: Frequency = 0.25Hz

Amplitude	Phase	Range
9.326E-07	1.0537897	1.0223509
9.396E-07	1.0531537	1.0199999
9.691E-07	1.0598287	1.0099969
0.000001011	1.0608657	0.9999949
0.000001066	1.0472523	0.9899979
0.000001099	1.0363947	0.98
0.000001124	1.0429895	0.9699973
0.000001165	1.0482836	0.9599998
0.000001215	1.0447771	0.9499967
0.000001243	1.043826	0.9400001
0.000001251	1.0493493	0.9299983
0.000001257	1.0521396	0.9199996
0.000001272	1.0399986	0.9099979
0.000001304	1.028952	0.8999994
0.000001336	1.0437171	0.8899984
0.000001356	1.0635529	0.8799931
0.000001367	1.0559211	0.8700007
0.000001412	1.0430582	0.8600001
0.000001479	1.0397121	0.8499984
0.00000153	1.0328939	0.8400013
0.000001573	1.034481	0.8299975
0.000001631	1.0510567	0.8200029
0.00000167	1.0670479	0.8099982
0.000001674	1.0796702	0.8000003
0.00000168	1.0895251	0.7899996
0.000001696	1.0949796	0.78
0.00000171	1.1069373	0.7699962
0.000001733	1.1246932	0.7600009
0.000001801	1.1416814	0.7500015
0.000001887	1.1658431	0.7399987
0.000001944	1.1957916	0.7300009
0.000002018	1.2201251	0.7199999
0.000002127	1.2378581	0.7099989
0.00000226	1.2645809	0.6999999
0.000002375	1.2917333	0.6900019
0.000002496	1.3139756	0.6799988
0.000002673	1.3439413	0.669998
0.000002887	1.366808	0.6599962
0.000003101	1.3747607	0.6499992
0.000003362	1.394585	0.6399966
0.000003652	1.4161511	0.6299981
0.000003954	1.4228433	0.6199972
0.00000428	1.4317356	0.6099999
0.000004676	1.453147	0.5999971
0.000005268	1.4713613	0.5899983